SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 2 Radian measure

Exercise 2A

Question 1

$$\frac{50}{360}\times 2\times\pi\times 12.4$$
$$=10.8\,\text{cm}$$

Question 2

$$\frac{235}{360}\times 2\times\pi\times 14.7$$
$$=60.3\,\text{cm}$$

Question 3

$$\frac{70}{360}\times 2\times\pi\times 6.7$$
$$=8.2\,\text{cm}$$

Question 4

$$\frac{240}{360}\times 2\times\pi\times 8$$
$$=\frac{32\pi}{3}\,\text{cm}$$

Question 5

$$\frac{150}{360} \times 2 \times \pi \times 10$$
$$= \frac{25\pi}{3} \text{cm}$$

Question 6

$$\frac{280}{360} \times 2 \times \pi \times 6$$
$$= \frac{28\pi}{3} \text{ cm}$$

Question 7

$$\frac{1}{6} \times \pi \times 12^2$$
$$= 24\pi \text{ cm}^2$$

Question 8

$$\frac{110}{360} \times \pi \times 6^2$$
$$= 11\pi \text{ cm}^2$$

Question 9

$$\frac{2}{3} \times \pi \times 8^2$$
$$= \frac{128\pi}{3} \text{cm}^2$$

Question 10

$$\frac{155}{360} \times \pi \times 15.4^2$$
$$= 321 \text{ cm}^2$$

Question 11

$$\frac{17.6}{2 \times \pi \times 12.3} \times \pi \times 12.3^2$$
$$= 108 \text{ cm}^2$$

Question 12

$$\frac{(2 \times \pi \times 10 - 20)}{2 \times \pi \times 10} \times \pi \times 10^2$$
$$= 214 \text{ cm}^2$$

Question 13

$$\frac{100}{360}\pi(15)^2 - \frac{1}{2}(15)^2 \sin 100$$
$$= 86 \text{ cm}^2$$

Question 14

Sector area : $\frac{16}{20\pi} \times \pi(10)^2 = 80 \text{ cm}^{22}$

Central angle : $\frac{16}{20\pi} \times 360 = 91.7°$

Triangle area : $\frac{1}{2} \times 10^2 \times \sin 91.7° = 50 \text{ cm}^2$

$\therefore$ area of segment $= 80 - 50 = 30 \text{ cm}^2$

Question 15

Fraction of circle : $\frac{90}{100\pi} = \frac{9}{10\pi}$

Central angle : $\frac{9}{10\pi} \times 360° = 103°$

$\therefore$ area of triangle : $\frac{1}{2} \times 10^2 \times \sin 103° = 48.7 \text{ cm}^2$

Shaded area : $90 - 48.7 \approx 41 \text{ cm}^2$

Question 16

Triangle area : $\frac{1}{2} \times \sin 60° \times 12^2 = 36\sqrt{3}$

Sector area : $\frac{1}{2} \times \pi \times 12^2 = 24\pi$

Shaded area : $24\pi - 36\sqrt{3} = 12(2\pi - 3\sqrt{3}) \text{ cm}^2$

Question 17

Sector area : $\frac{135}{360} \times \pi \times 6^2 = \frac{27\pi}{2}$

Triangle area : $\frac{1}{2} \times 6^2 \times \sin 135° = 9\sqrt{2}$

Shaded area : $\frac{27\pi}{2} - 9\sqrt{2} = 9(\frac{3\pi}{2} - \sqrt{2}) \text{ cm}^2$

Question 18

Segment area : $\frac{150}{360} \times \pi(10)^2 = \frac{125\pi}{3} \text{ cm}^2$

Triangle area : $\frac{1}{2} \times 10^2 \times \sin 150° = 25 \text{ cm}^2$

$\therefore$ Shaded area $= \frac{125\pi}{3} - 25 = 25(\frac{5\pi}{3} - 1) \text{ cm}^2$

Question 19

a $\frac{112}{360} \times 2\pi \times 15.2 = 29.7 \text{ cm}$

b $\frac{248}{360} \times 2\pi \times 15.2 = 65.8 \text{ cm}$

Question 20

$$\frac{75}{360} \times 2\pi r = 24$$

$$r = \frac{24 \times 360}{75 \times 2\pi}$$

$$= 18.3 \text{ cm}$$

Question 21

$$\frac{50}{360} \times \pi \times 15^2 = 98.2 \text{ cm}^2$$

Question 22

Sector area : $\frac{140}{360} \times \pi \times 18^2 = 395.8 \text{ cm}^2$

Triangle area : $\frac{1}{2} \times 18^2 \times \sin 140° = 104.1 \text{ cm}^2$

Segment area : $395.8 - 104.1 \approx 292 \text{ cm}^2$

Question 23

$$\frac{378}{\pi \times 12^2} = \frac{\theta}{360} \quad (\theta > 180°)$$

$$\theta = \frac{378}{\pi \times 144} \times 360$$
$$= 301°$$

Acute angle $= 59°$

Question 24

$$\cos\theta = \frac{12^2 + 12^2 - 10^2}{2 \times 12 \times 12}$$

$$\theta = 49°$$

Sector area : $\frac{49}{360} \times \pi \times 12^2 = 61.6 \text{ cm}^2$

Triangle area : $\frac{1}{2} \times 12^2 \times \sin 49° = 54.3 \text{ cm}^2$

Segment area : $61.6 - 54.3 = 7.3 \text{ cm}^2$

Question 25

The hour hand would travel 15° in 30 minutes.

$$\frac{15}{360} \times 2 \times \pi \times 8 = \frac{2\pi}{3} \text{ cm}$$

The minute hand would travel 180° in 30 minutes.

$$\frac{1}{2} \times 2\pi \times 12 = 12\pi \text{ cm}$$

Question 26

$3° = 180'(180 \text{ minutes})$

$\therefore 180$ nautical miles travelled

$$\frac{3}{360} \times 2\pi \times 6350 = 332.485 \text{ km}$$

$$180\text{nm} = 332.485 \text{ km}$$

$$1 \text{ nm} = \frac{332.485}{180}$$

$$= 1.85 \text{ km}$$

Question 27

Major arc length becomes the circumference of the hat

$$\frac{240}{360} \times 2\pi \times 10 = \frac{40\pi}{3} = 41.89 \text{ cm}$$

$$2\pi r = \frac{40\pi}{3}$$

$$r = \frac{40\pi}{2\pi \times 3}$$

$$= 6\tfrac{2}{3} \text{ cm}$$

$$h^2 = 10^2 - (6\tfrac{2}{3})^2$$

$$= \frac{500}{9}$$

$$h = \frac{10\sqrt{5}}{3} \text{ cm}$$

Exercise 2B

Question 1

Arc length = 3 × radius $\Longrightarrow \theta = 3$ rads

Question 2

Arc length = 1.5 × radius $\Longrightarrow \theta = 1.5$ rads

Question 3

Arc length = 5 × radius $\Longrightarrow \theta = 5$ rads

Question 4

Arc length = 2.5 × radius $\Longrightarrow \theta = 2.5$ rads

Question 5

Arc length = 4 x radius $\Longrightarrow \theta = 4$ rads

Question 6

Arc length = 4 × radius $\Longrightarrow \theta = 4$ rads

Question 7

$180° = \pi$ radians

$90° = \frac{\pi}{2}$ radians

Question 8

$180° = \pi$ radians

$30° = \frac{\pi}{6}$ radians

Question 9

$180° = \pi$ radians

$150° = \frac{5\pi}{6}$ radians

Question 10

$180° = \pi$ radians

$135° = \frac{3\pi}{4}$ radians

Question 11

$180° = \pi$ radians

$5° = \frac{\pi}{36}$ radians

Question 12

$180° = \pi$ radians

$18° = \frac{\pi}{10}$ radians

Question 13

$180° = \pi$ radians

$80° = \frac{4\pi}{9}$ radians

Question 14

$180° = \pi$ radians

$130° = \frac{13\pi}{18}$ radians

Question 15

$\frac{\pi}{4}$ radians $= \left(\frac{180}{4}\right)° = 45°$

Question 16

$\frac{\pi}{3}$ radians $= \left(\frac{180}{3}\right)° = 60°$

Question 17

$\frac{2\pi}{3}$ radians $= \left(\frac{2 \times 180}{3}\right)° = 120°$

Question 18

π radians $= 180°$

Question 19

$\frac{\pi}{12}$ radians $= \left(\frac{180}{12}\right)° = 15°$

Question 20

$\frac{\pi}{5}$ radians $= \left(\frac{180}{5}\right)° = 36°$

Question 21

$$\frac{7\pi}{36} \text{ radians} = \left(\frac{7 \times 180}{36}\right)° = 35°$$

Question 22

$$\frac{7\pi}{18} \text{ radians} = \left(\frac{7 \times 180}{18}\right)° = 70°$$

Question 23

$$\frac{32\pi}{180} \text{ radians} = 0.56 \text{ radians}$$

Question 24

$$\frac{63\pi}{180} \text{ radians} = 1.10 \text{ radians}$$

Question 25

$$\frac{115\pi}{180} \text{ radians} = 2.01 \text{ radians}$$

Question 26

$$\frac{170\pi}{180} \text{ radians} = 2.97 \text{ radians}$$

Question 27

$$\frac{16\pi}{180} \text{ radians} = 0.28 \text{ radians}$$

Question 28

$\frac{84\pi}{180}$ radians $= 1.47$ radians

Question 29

$\frac{104\pi}{180}$ radians $= 1.82$ radians

Question 30

$\frac{26\pi}{180}$ radians $= 0.45$ radians

Question 31

$1.5 \times \frac{180}{\pi} = 86°$

Question 32

$2.3 \times \frac{180}{\pi} = 132°$

Question 33

$1.4 \times \frac{180}{\pi} = 80°$

Question 34

$0.6 \times \frac{180}{\pi} = 34°$

Question 35

$\frac{1}{\sqrt{2}}$

Question 36

$\frac{1}{2}$

Question 37

$-\frac{1}{\sqrt{2}}$

Question 38

1

Question 39

$\frac{\sqrt{3}}{2}$

Question 40

$\frac{1}{\sqrt{2}}$

Question 41

$\frac{1}{\sqrt{2}}$

Question 42

$-\sqrt{3}$

Question 43

0

Question 44

Undefined

Question 45

$-\frac{1}{2}$

Question 46

$-\frac{1}{\sqrt{3}}$

Question 47

$-\frac{\sqrt{3}}{2}$

Question 48

0

Question 49

$\frac{1}{2}$

Question 50

0

Question 51

0.84

Question 52

–0.42

Question 53

–0.75

Question 54

0.14

Question 55

0.83

Question 56

0.99

Question 57

3.60

Question 58

0.75

Question 59

0.20 radians

Question 60

1.37 radians

Question 61

0.34 radians

Question 62

1.04 radians

Question 63

a 3 revolutions/s

$= 3 \times 2\pi \text{ / s}$

$= 6\pi$ radians/s

b 15 revolutions/min

$= \frac{1}{4}$ revolutions/s

$= \frac{1}{4} \times 2\pi$ radians/s

$= \frac{\pi}{2}$ radians/s

c 90°/s

$= \frac{1}{4}$ revolutions/s

$= \frac{1}{4} \times 2\pi$ radians/s

$= \frac{\pi}{2}$ radians/s

Question 64

a 2π radians/min

$=1$ revolution/min

b $\frac{3\pi}{4}$ radians/s $= \frac{3\pi}{4\times 2\pi}$ revolutions/s

$= \frac{3}{8}$ revolutions/s

$= \frac{3}{8} \times 60$ revolutions/minute

$= 22.5$ revolutions/minute

c $\frac{\pi}{3}$ radians/s

$= \frac{\pi}{3\times 2\pi}$ revolutions/s

$= \frac{1}{6}$ revolutions/s

$= \frac{1}{6} \times 60$ revolutions/minute

$=10$ revolutions/minute

Question 65

$$\sin 1 = \frac{6}{x}$$

$$x = \frac{6}{\sin 1}$$

$$= 7.1\,\text{cm}$$

Question 66

$$\tan 1.2 = \frac{8}{x}$$

$$x = \frac{8}{\tan 1.2}$$

$$= 3.1\,\text{cm}$$

Question 67

$$\sin 0.6 = \frac{h}{20}$$
$$h = 20 \times \sin 0.6$$
$$= 11.3$$
$$x^2 = 11.3^2 + 6^2$$
$$x = 12.8 \text{ cm}$$

Question 68

$$\frac{x}{\sin 1.1} = \frac{15}{\sin 1.8}$$
$$x = \frac{14 \times \sin 1.1}{1.8}$$
$$= 12.8 \text{ cm}$$

Question 69

Supplementary angle : $\pi - 0.64 = 2.50$

$$x^2 = 10^2 + 7^2 - 2 \times 7 \times 10 \times \cos 2.5$$
$$x = 16.2 \text{ cm}$$

Question 70

$$\cos x = \frac{6.1^2 + 5.0^2 - 7.2^2}{2 \times 6.1 \times 5.0}$$
$$x = 1.4 \text{ radians}$$

Question 71

a $15 \text{ minutes} = \frac{1}{4} \text{ revolutions}$

$1 \text{ revolution } = 2\pi \text{ radians}$

$\frac{1}{4} \times 2\pi = \frac{\pi}{2} \text{radians}$

b $40 \text{ minutes} = \frac{2}{3} \text{ revolutions}$

$\frac{2}{3} \times 2\pi = \frac{4\pi}{3} \text{radians}$

c $50 \text{ minutes} = \frac{5}{6} \text{ revolutions}$

$\frac{5}{6} \times 2\pi = \frac{5\pi}{3} \text{ radians}$

d $55 \text{ minutes} = \frac{11}{12} \text{ revolutions}$

$\frac{11}{12} \times 2\pi = \frac{11\pi}{6} \text{ radians}$

Question 72

$100 \text{ grads} = 90° = \frac{\pi}{2} \text{ radians}$

a $50 \text{ grads} = \frac{\pi}{4} \text{ radians}$

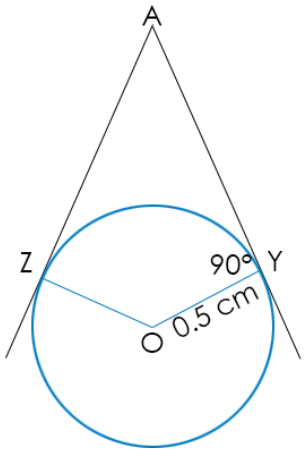
b $75 \text{ grads} = \frac{3}{4} \times \frac{\pi}{2} = \frac{3\pi}{8} \text{ radians}$

c $10 \text{ grads} = \frac{1}{10} \times \frac{\pi}{2} = \frac{\pi}{20} \text{ radians}$

d $130 \text{ grads} = \frac{13}{10} \times \frac{\pi}{2} = \frac{13\pi}{20} \text{ radians}$

Question 73

a Assume the pipe has a diameter of 1 cm, (radius 0.5 cm).



Points of contact Y and Z are tangent to pipe forming right angles.

We need to find the length of AY given $\angle OAY = 0.5$ radian.

$$\tan 0.5 = \frac{0.5}{AY}$$
$$AY = 0.5 \div \tan 0.5$$
$$= 0.915 \text{ cm}$$

Each marking should be made 0.915 cm along the guide.

b yes.

If $\angle BAC = \frac{\pi}{2}$ radians, $\angle OAY = \frac{\pi}{4}$ radians and consequently $\angle YOA = \frac{\pi}{4}$ radians.

ΔOYA is then isosceles, with OY = AY.

The calibrations should then be placed 0.5 cm apart which is a simpler scale.

Exercise 2C

Question 1

$$\begin{aligned} l &= r\theta \\ &= 5 \times 0.8 \\ &= 4 \text{ cm} \end{aligned}$$

Question 2

$$\begin{aligned} l &= r\theta \\ &= 10 \times 2.5 \\ &= 25 \text{ cm} \end{aligned}$$

Question 3

$$\begin{aligned} l &= r\theta \\ &= 7.8 \times (2\pi - 4.5) \\ &= 13.9 \text{ cm} \end{aligned}$$

Question 4

$$\begin{aligned} A &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 4^2 \times 1 \\ &= 8 \text{ cm}^2 \end{aligned}$$

Question 5

$$\begin{aligned} A &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 6^2 \times 2.5 \\ &= 45 \text{ cm}^2 \end{aligned}$$

Question 6

$$\begin{aligned} A &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 10^2 \times (2\pi - 4) \\ &= 114 \text{ cm}^2 \end{aligned}$$

Question 7

$$\begin{aligned} A &= \frac{1}{2} r^2 (\theta - \sin\theta) \\ &= \frac{1}{2} \times 59^2 \times (1 - \sin 1) \\ &= 276 \text{ cm}^2 \end{aligned}$$

Question 8

$$\begin{aligned} A &= \frac{1}{2} r^2 (\theta - \sin\theta) \\ &= \frac{1}{2} \times 5.1^2 \times ((2\pi - 3.5) - \sin(2\pi - 3.5)) \\ &= 31.6 \text{ cm}^2 \end{aligned}$$

Question 9

$$\begin{aligned} A &= \frac{1}{2} r^2 (\theta - \sin\theta) \\ &= \frac{1}{2} \times 7.5^2 \times (2.2 - \sin 2.2) \\ &= 39.1 \text{ cm}^2 \end{aligned}$$

Question 10

$$\begin{aligned} l &= r\theta \\ &= 15 \times 1.2 \\ &= 18 \text{ cm} \end{aligned}$$

Question 11

a $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 15^2 \times 0.8$

$= 90 \text{ cm}^2$

b $\pi(15)^2 - 90 = 617 \text{ cm}^2$

Question 12

a $l = r\theta$

$= 8 \times 1$

$= 8 \text{ cm}$

b $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 8^2 \times (1 - \sin 1)$

$= 5.1 \text{ cm}^2$

Question 13

a $A = \frac{1}{2} r^2 \theta$

$15 = \frac{1}{2} \times 5^2 \times \theta$

$\theta = \frac{15}{12.5}$

$= 1.2 \text{ radians}$

$l = r\theta$

$= 1.2 \times 5$

$= 6 \text{ cm}$

b $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 5^2 \times (1.2 - \sin(1.2))$

$= 3.35 \text{ cm}^2$

Question 14

$l = r\theta$

$20 = 8 \times \theta$

$\theta = 2.5 \text{ radians}$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 8^2 \times 2.5$

$= 80 \text{ cm}^2$

Question 15

$$A = \frac{1}{2} r^2 \theta$$
$$9 = \frac{1}{2} \times 6^2 \times \theta$$
$$\theta = 0.5 \text{ radians}$$

$$A = \frac{1}{2} r^2 (\theta - \sin\theta)$$
$$= \frac{1}{2} \times 6^2 \times (0.5 - \sin(0.5))$$
$$= 0.37 \text{ cm}^2$$

Question 16

Area of sector OBC Area of sector OAD

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 12^2 \times 1.5$$
$$= 108 \text{ cm}^2$$

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 6^2 \times 1.5$$
$$= 27 \text{ cm}^2$$

$\therefore$ required area $108 - 27 = 81 \text{cm}^2$

Question 17

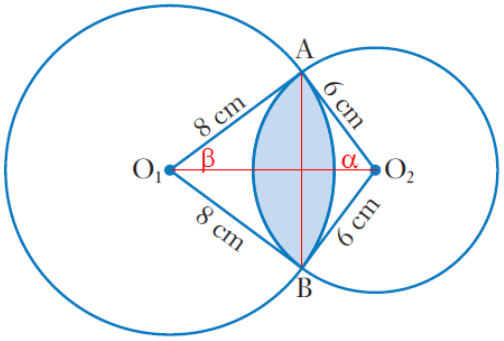
$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 9^2 \times 3$$
$$= 121.5 \text{ cm}^2$$

$$A = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 5^2 \times 3$$
$$= 37.5 \text{ cm}^2$$

$\therefore$ required area $121.5 - 37.5 = 84 \text{ cm}^2$

Question 18

The total shaded area is the sum of segments O_1AB and O_2AB.



In $\Delta O_1AO_2, \angle O_1AO_2 = 90°$,

$$\tan \alpha = \frac{8}{6}$$
$$\alpha = 0.9273$$
$$\angle AO_2B = 1.8456$$
$$\angle AO_1B = \pi - 1.8546 = 1.2870$$

Segment O_2AB

$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$
$$= \frac{1}{2} 6^2 (1.8546 - \sin(1.8456))$$
$$= 16.01 \text{ cm}^2$$

Segment O_1AB

$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$
$$= \frac{1}{2} 8^2 (1.2870 - \sin(1.2870))$$
$$= 10.46 \text{ cm}^2$$

$\therefore$ required area $16.10 - 10.46 = 26.6 \text{ cm}^2$

Question 19

Total area = segment OAD + (sector OCB - sector OAD)

Sector ABCD

$$\frac{1}{2}8^2(0.8) - \frac{1}{2}5^2(0.8) = 15.6 \text{ cm}^2$$

Segment in OAD

$$\frac{1}{2}5^2(0.8 - \sin(0.8)) = 1.0 \text{ cm}^2$$

Total area $= 16.6 \text{ cm}^2$

Question 20

Area of quadrilateral OACB: $0.5 \times 6 \times 8 \times 2 = 48 \text{cm}^2$

$$\tan \angle \text{AOC} = \frac{8}{6}$$
$$\angle \text{AOC} = 0.93$$

Area of sector OAB

$$A = \frac{1}{2} \times 6^2 \times (0.93 \times 2)$$
$$= 33.4 \text{ cm}^2$$

$\therefore$ required area : $48\text{-}33.4 = 14.6 \text{ cm}^2$

Question 21

Area of sector OABCD : $A = \frac{1}{2} \times 5^2 \times 2$

$= 25 \text{ cm}^2$

Area of ΔOAD : $A = \frac{1}{2} \times 5^2 \sin 2$

$= 11.37 \text{ cm}^2$

Area of segment cur off by chord BC : $A = \frac{1}{2} 5^2 (1 - \sin 1)$

$= 1.98 \text{ cm}^2$

Required area $= 25 - 11.37 - 1.98 = 11.65 \text{ cm}^2$

Question 22

a $l = r\theta$

$= 75 \times 0.8$

$= 60 \text{ cm}$

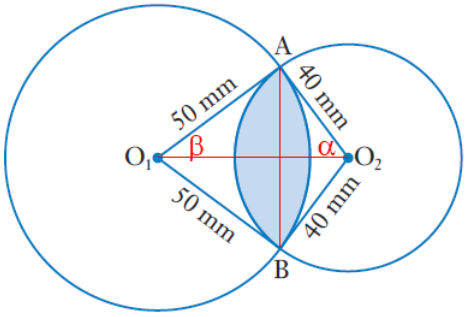
distance travelled : 120 cm

b $\text{BC}^2 = 75^2 + 75^2 + 2 \times 75 \times 75 \times \cos 0.8$

$\text{BC} = 58.4 \text{ cm}$

Difference of 1.6 cm

Question 23



$$\cos B = \frac{50^2 + 70^2 - 40^2}{2 \times 50 \times 70}$$

$$\beta = 0.59$$

$$\angle AO_1B = 1.18$$

$$A = \frac{1}{2} \times 50^2 \times (1.18 - \sin(1.18))$$

$$= 309 \text{ mm}^2$$

$$\cos \alpha = \frac{40^2 + 70^2 - 50^2}{2 \times 40 \times 70}$$

$$\alpha = 0.78$$

$$\angle AO_2B = 1.56$$

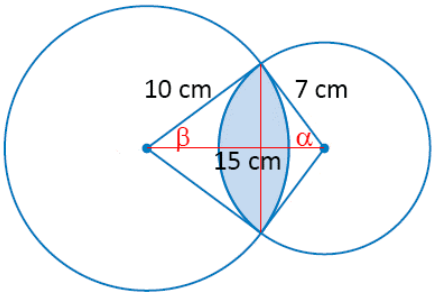
$$A = \frac{1}{2} \times 40^2 \times (1.56 - \sin(1.56))$$

$$= 448 \text{ mm}^2$$

Total Area : $319 + 448 = 767 \text{ mm}^2$

Area, to nearest $10 \text{ mm}^2 = 770 \text{ mm}^2$

Question 24



$$\cos \alpha = \frac{10^2 + 15^2 - 7^2}{2 \times 10 \times 15}$$

$$\alpha = 0.403$$

$$\cos \beta = \frac{7^2 + 15^2 - 10^2}{2 \times 7 \times 15}$$

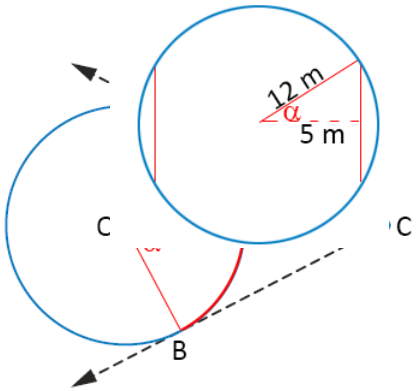
$$\beta = 0.594$$

$$\text{Arc } 1 = 10 \times (0.403 \times 2) = 8.06$$

$$\text{Arc } 2 = 7 \times (0.594 \times 2) = 8.32$$

$$\therefore \text{ perimeter } = 16.4 \text{ cm}$$

Question 25



In $\Delta OAC, \angle OAC = 90°$ (tangent-radius)

$$\cos\alpha = \frac{10}{22}$$

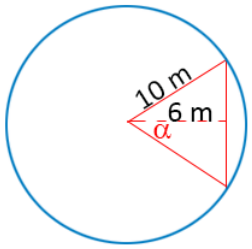
$$\alpha = 1.10$$

Arc length $AB = 10 \times 2.2 = 22\,\text{cm}$

Percentage of circumference

$$\frac{22}{2 \times \pi \times 10} \times 100 = 35\%$$

Question 26



$$\cos\alpha = \frac{6}{10}$$

$$\alpha = 0.927$$

Area of segment

$$\frac{1}{2} \times 10^2 (1.854 - \sin(1.854))$$

$$= 44.69\ \text{m}^2$$

$$\therefore \text{required area} = \pi \times 10^2 - 44.69 = 269\ \text{m}^2$$

Question 27

$$\cos\alpha = \frac{5}{12}$$

$$\alpha = 1.4$$

Area of segment

$$\frac{1}{2} \times 12^2 \times (2.28 - \sin 2.28)$$

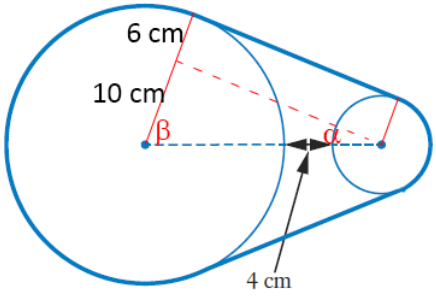
$$= 109.5$$

Area available to goat

$$\pi 12^2 - 2 \times 109.5 = 233\ \text{m}^2$$

Question 28

Distance between centres: 26 cm



Length of the belt connecting wheels (dotted length)

$l^2 = 26^2 - 10^2$

$l = 24$

$\tan\beta = \frac{24}{10}$

$\beta = 1.176$

Angle subtended by belt on the large wheel is $2\pi - 2 \times 1.176 = 3.93$ radians

Length of belt in contact with large wheel : $3.93 \times 16 = 62.90$ cm

$\alpha = \frac{\pi}{2} - 1.176 = 0.395$

Angle subtended by belt on the small wheel:

$2\pi - (\frac{\pi}{2} + 0.395) = 2.352$

Length of belt in contact with small wheel

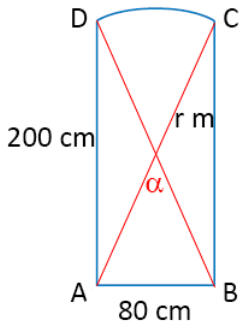
$2.352 \times 6 = 14.11$ cm

$\therefore$ Total belt length

$62.90 + 14.11 + 24 \times 2$

$= 125$ cm

Question 29



Radius of circular top

$r^2 = 40^2 + 100^2$

$r = 107.7 \text{ cm}$

$\cos\alpha = \dfrac{11600 \times 2 - 80^2}{2 \times 11600}$

$\alpha = 0.761$

Area of segment at the top of the door

$A = \dfrac{1}{2} r^2(\theta - \sin\theta)$

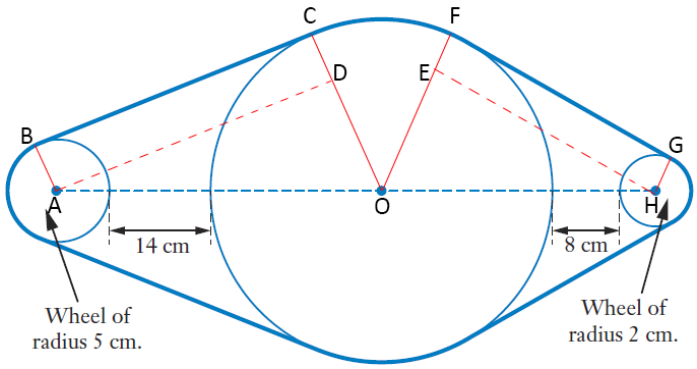
$= \dfrac{1}{2} \times 11600 \times (0.761 - \sin(0.761))$

$= 414 \text{ cm}^2$

$\therefore$ Area of door $= 200 \times 80 + 414 = 16\ 410 \text{ cm}^2$

Question 30

Consider AOH to be a line of symmetry



$\text{In } \Delta AOD, AO = 39\,\text{cm}, \angle ADO = 90°, OD = 15\,\text{cm}$

$AD^2 = 39^2 - 15^2$

$AD = 36\,\text{cm}$

$\sin\angle DAO = \frac{15}{39} = 0.395$

$\angle DOA = \pi - \frac{\pi}{2} - 0.395 = 1.966$

$\text{In } \Delta OEH, OE = 18\,\text{cm}, \angle OEH = 90°, OH = 30\,\text{cm}$

$EH^2 = 30^2 - 18^2$

$EH = 24\,\text{cm}$

$\sin\angle EHO = \frac{18}{30} = 0.644$

$\angle EOH = \frac{\pi}{2} - 0.644 = 0.927$

We need to determine the arc lengths from each of the three wheels.

The angle subtended by the belt on the 2 cm wheel, above the symmetry line,

$\pi - \frac{\pi}{2} - 0.644 = 0.927$

Length of belt in contact, above line AOH=$2 \times 0.927 = 1.85\,\text{cm}$

The angle subtended by the belt on the 5 cm wheel, above the symmetry line = 1.176

Length of belt in contact, above line AOH = $1.176 \times 5 = 5.88\,\text{cm}$

The angle subtended by the belt on the 20 cm wheel, above line AOH,
$\pi - 1.176 - 0.927 = 1.039$

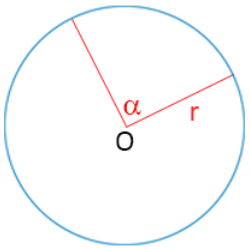
The length of the belt in contact, above line AOH $= 1.039 \times 20 = 20.77\,\text{cm}$

The length of belt in the top half $= 5.88 + 36 + 20.77 + 24 + 1.85 = 88.5\,\text{cm}$

Total belt length $88.5 \times 2 = 177\,\text{cm}$

Question 31

a



$$A = \frac{1}{2}r^2\theta = 10$$

$$P = r\theta + 2r = 14 \Rightarrow r = \frac{14}{\theta + 2}$$

Subsitution for r in the area equation produces

$$\frac{1}{2}\left(\frac{14}{\theta+2}\right)^2\theta = 10$$

By classpad, or as shown below, $\theta = 0.8$ or 5

$$\frac{196\theta}{\theta^2 + 4\theta + 4} = 20$$

$$196\theta = 20(\theta^2 + 4\theta + 4)$$

$$20\theta - 116\theta + 80 = 0$$

$$4(\theta - 5)(5\theta - 4) = 0$$

$$\theta = 5 \text{ or } \theta = 0.8$$

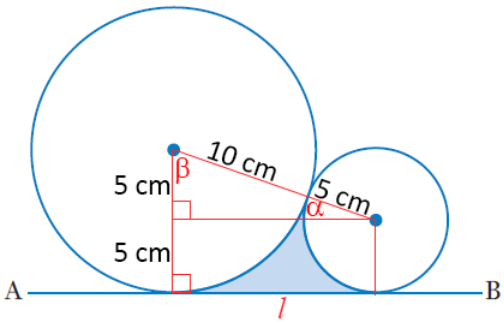
If the sector is minor, $\theta < \pi \therefore$ we can disregard $\theta = 5$ as a solution.

GIven $\theta = 0.8$, $r = \frac{14}{0.8 + 2} = 5.$

The radius of the circle is 5 cm.

b A major sector has $\theta > \pi,$ so from the previous question $\theta = 5$ which produces $r = 2$ cm

Question 32



$$\sin\alpha = \frac{5}{15}$$
$$\alpha = 0.34$$
$$\beta = \frac{\pi}{2} - 0.34 = 1.23$$

Distance between vertical radii: $l^2 = 15^2 - 5^2$

$$l = \sqrt{200} = 10\sqrt{2}$$

Area enclosed by trapezium: $\frac{1}{2}(10+5)\times 10\sqrt{2} = 75\sqrt{2}$

Area of small sector: $\frac{1}{2}\times 5^2 \times (0.34 + \frac{\pi}{2}) = 23.9$

Area of large sector: $\frac{1}{2}\times 10^2 \times 1.23 = 61.5$

Shaded area: $75\sqrt{2} - 23.9 - 61.5 = 20.7\,\text{cm}^2$

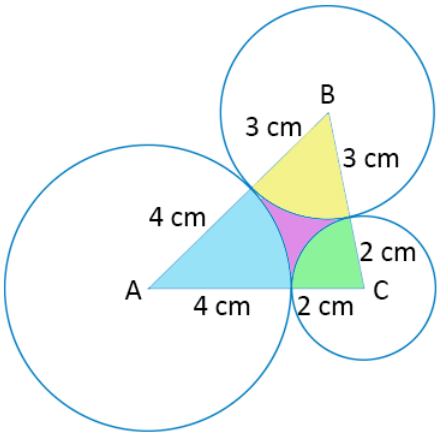
Question 33

$$\cos\angle BAC = \frac{7^2+6^2-5^2}{2\times 7\times 6} \qquad \cos\angle ABC = \frac{7^2+5^2-6^2}{2\times 7\times 5}$$

$$\angle BAC = 0.775 \qquad \angle ABC = 0.997$$

$$\therefore \angle ACB = \pi - 0.775 - 0.997$$

$$= 1.370$$



Area of triangle ABC : $0.5\times 7\times 6\times \sin 0.775 = 14.69$

Area of sector in circle centre A (shaded blue): $0.5\times 4^2\times 0.775 = 6.20$

Area of sector in circle centre B (shaded yellow): $0.5\times 3^2\times 0.997 = 4.49$

Area of sector in circle centre C (shaded green): $0.5\times 2^2\times 1.370 = 2.74$

Area of triangle outside of circle (shaded pink): $14.69 - (6.20 + 4.49 + 2.74) = 1.26$

Percentage of triangle : $\frac{1.26}{14.69}\times 100\% = 8.6\%$

Question 1

a $(x+3)(2x-1) = 2x^2 - x + 6x - 3$
$= 2x^2 + 5x - 3$

b $(x+7)(3x-4) = 3x^2 - 4x + 21x - 28$
$= 3x^2 + 17x - 28$

c $(x+5)(x-1)(x+3) = (x+5)(x^2 + 2x - 3)$
$= x^3 + 2x^2 - 3x + 5x^2 + 10x - 15$
$= x^3 + 7x^2 + 7x - 15$

d $(2x+1)(x-3)(x-2) = (x-2)(2x^2 - 5x - 3)$
$= 2x^3 - 5x^2 - 3x - 4x^2 + 10x + 6$
$= 2x^3 - 9x^2 + 7x + 6$

Question 2

a $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

c $\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

d $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

e $\frac{1}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{9-5}$
$= \frac{3-\sqrt{5}}{4}$

f

$$\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{9-2}$$
$$= \frac{3+\sqrt{2}}{7}$$

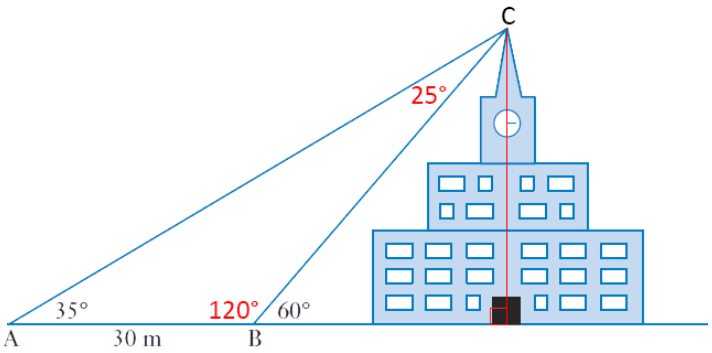
g

$$\frac{2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{2(1-\sqrt{5})}{1-5}$$
$$= \frac{2(1-\sqrt{5})}{-4}$$
$$= \frac{\sqrt{5}-1}{2}$$

h

$$\frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$
$$= \frac{3(\sqrt{5}-\sqrt{2})}{3}$$
$$= \sqrt{5}-\sqrt{2}$$

Question 3



In ΔABC

$$\frac{\sin 25°}{30} = \frac{\sin 35°}{BC}$$
$$BC = \frac{30 \sin 35°}{\sin 25°}$$
$$= 40.7$$

$$\sin 60° = \frac{h}{40.7}$$
$$h = 40.7 \times \sin 60°$$
$$= 35 \text{ m (nearest m)}$$

Question 4

a 1 revolution in 5 seconds

$\Rightarrow \frac{2\pi}{5}$ in 1 second

$l = r\theta$

$= 1.8 \times \frac{2\pi}{5}$

$= 2.26\,\text{m}$

b $l = r\theta$

$= \frac{2\pi}{5} \times 1$

$= 1.26\,\text{m}$

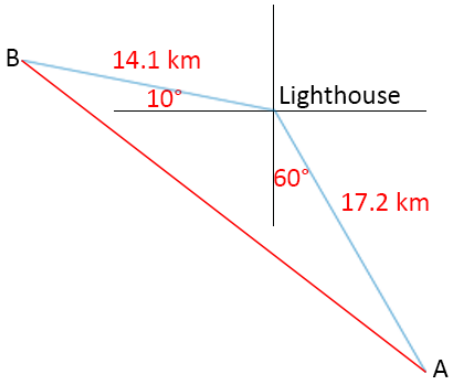
Question 5

$\text{AB}^2 = 14.1^2 + 17.2^2 - 2 \times 14.1 \times 17.2 \times \cos 160°$

$\text{AB} = 30.8\,\text{km}$

$\frac{\sin \angle \text{BAL}}{14.1} = \frac{\sin 160°}{30.8}$

$\angle \text{BAL} = 9°$



B is 30.8 km from A on a bearing of N 69° W

Question 6

$$\text{Area of } \Delta BCD = 0.5 \times 73.2 \times 87.8 \times \sin 67°$$
$$= 2958 \text{ m}^2$$

$$BD^2 = 73.2^2 + 87.8^2 - 2 \times 73.2 \times 87.8 \times \cos 67°$$
$$BD = 89.7 \text{ m}$$

$$\frac{\sin 82°}{89.7} = \frac{\sin \angle ADB}{67.2}$$
$$\angle ADB = 48°$$
$$\angle ABD = 50°$$

$$\frac{\sin 50°}{AD} = \frac{\sin 82°}{89.7}$$
$$AD = 69.4 \text{ m}$$

$\therefore$ Area ΔABD
$$= 0.5 \times 67.2 \times 69.4 \times \sin 82°$$
$$= 2309 \text{ m}^2$$
$$\text{Total area } = 2958 + 2309 = 5267 \text{ m}^2$$
$$\text{Total Perimeter} = 73.2 + 87.8 + 67.2 + 69.4 = 297.6 \text{ m}$$